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# Absence of "Condensed Matter Effects" in the Solar Neutrino Problem

**John N. Bahcall[1] and Andrew Gould[2]**

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It has been suggested by Kim, Rabinowitz, and Yoon that the solar neutrino problem can be explained by taking account of so-called "condensed matter effects." We show with the aid of Liouville's theorem that the effects in question are already included in the standard treatment.

## 1. INTRODUCTION

Kim *et al.* (1993) assert that the solar neutrino problem [i.e., the discrepancy between the observed and predicted flux of neutrinos from the sun (Bahcall and Davis, 1976)] can be resolved by taking account of what they call "condensed matter effects." Specifically, they argue that in the dense environment of the sun, the phase-space densities of fusing nuclei are lower at the point where they last scatter off ambient particles (prior to reacting) than are the phase-space densities of the same particles when they are infinitely far apart. Since conventional calculations are based on the latter densities, Kim *et al.* argue that the standard calculations overestimate the event rate.

In the words of Kim *et al.*, "The problem may be viewed with cautioned analogy to that of molecules in a gravitational potential where in the ideal case an altitude increase results only in a number density decrease with the maintenance of an isothermal Maxwellian distribution.... [Hence], the flux is reduced" (p. 1198).

Here we show, using Liouville's theorem, that the reduced phase-space density at close separations is *already* taken into account in the standard

[1]Institute for Advanced Study, Princeton, New Jersey 08540.
[2]Department of Astronomy, Ohio State University, Columbus, Ohio 43210. E-mail: gould@payne-mps.ohio-state.edu.



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calculations. Hence there is no reduction in the reaction rates due to this effect.

## 2. PHASE-SPACE DENSITIES WITH AND WITHOUT COLLISIONS

The basic argument of Kim *et al.* (1993) is that in the dense environment of the sun there is some radius $r_e$ around a nucleus outside of which other nuclei are driven to equilibrium by elastic collisions. The incident nuclei cannot be considered free particles until they reach this radius. Hence the calculation of reaction rates (which implicitly assumes free particles) should use the phase-space distribution of particles at $r_e$, rather than at infinity. The former is smaller than the latter by a factor $\exp[-\phi(r_e)/kT]$, where $\phi(r_e)$ is the potential energy at $r_e$ and $T$ is the ambient temperature. Since the conventional calculation uses the distribution at infinity, it must—according to Kim *et al.*—overestimate the event rates.

The fallacy of this argument is as follows: the standard treatment implicitly assumes that nuclei are collisionless. To find the correction due to collisions one should compare the actual distribution function at $r_e$ with the distribution function of a collisionless gas at the same radius. Alternatively, one might take the actual distribution at $r_e$, integrate the orbits backward to infinity assuming no collisions, and then compare the two distributions at infinity. By contrast, Kim *et al.* compare the actual distribution at $r_e$ with the collisionless distribution at infinity, a comparison that has no physical meaning.

How is the distribution at $r_e$ affected by collisions? Not at all. We first illustrate this fact for a particular case where one of the nuclei is very massive and hence is not moving relative to the plasma. We assume that the potential due to this nucleus $\phi(\mathbf{r})$ vanishes at infinity, but is otherwise completely arbitrary, $\phi(\infty) = 0$. We assume that the distribution function of the second species of nuclei $F(\mathbf{v}, \mathbf{r})$ is isotropic and homogeneous at infinity,

$$F(\mathbf{u}, \infty) = f(u) \tag{2.1}$$

We do not yet assume that $f$ is Maxwellian. Let us first suppose that the gas is collisionless. We can then use Liouville's theorem to evaluate the phase-space density at an arbitrary point in space $\mathbf{r}$ and for an arbitrary unbound velocity $\mathbf{v}$. Since energy is conserved over the orbit, the speed of the particle at infinity is $u = [v^2 + 2\phi(\mathbf{r})/m]^{1/2}$, where $m$ is the mass. Hence, since phase-space density is conserved (Liouville, 1837),

$$F(\mathbf{v}, \mathbf{r}) = f([v^2 + 2\phi(\mathbf{r})/m]^{1/2}) \tag{2.2}$$

That is, the distribution of unbound particles is isotropic, depending only on the magnitude of **v** and not its direction. (The distribution of bound particles is not constrained by this argument.) Now, we further assume that $f$ is Maxwellian,

$$f(u) = \frac{n_\infty}{(2\pi\sigma^2)^{3/2}} \exp\left(-\frac{u^2}{2\sigma^2}\right), \qquad \sigma^2 \equiv \frac{kT}{m} \tag{2.3}$$

where $T$ is the temperature and $n_\infty$ is the number density of the second species at infinity. Combining equations (2.2) and (2.3), we find

$$\begin{aligned} F(\mathbf{v}, \mathbf{r}) &= \frac{n_\infty}{(2\pi\sigma^2)^{3/2}} \exp\left(-\frac{v^2}{2\sigma^2}\right) \exp\left[-\frac{\phi(\mathbf{r})}{m\sigma^2}\right] \\ &= F(\mathbf{v}, \infty) \exp\left[-\frac{\phi(\mathbf{r})}{kT}\right] \end{aligned} \tag{24}$$

That is, the distribution function at **r** of a collisionless gas which is Maxwellian at infinity is also Maxwellian, but is suppressed by a standard Boltzmann factor. Note that equation (2.4) is exactly what one calculates for a *collisional* gas. There is no difference between the distribution functions for a collisional gas and a collisionless gas. Hence the interaction rates are not affected by using formulas appropriate for a collisionless gas.

We have gone to some trouble to prove this result explicitly for the special case when one of the reactants is very heavy. We have adopted this approach in order to allow the reader to see as clearly as possible the point at which the Kim *et al.* argument fails: namely, when they compare the distribution functions in the two cases at different spatial points instead of comparing them at the same point.

The same result holds generally and follows from the fact that the nuclei in the sun have everywhere maximal entropy distributions. The maximal entropy nature of these distributions is enforced by the fact that there are far more elastic than inelastic collisions in the sun, as noted by Kim *et al.* (1993). These distributions are exponential in the total (kinetic plus potential) energy of all species of particles concerned and are exactly the same whether the gas is collisional or collisionless.

## 3. DISCUSSIONS AND CONCLUSIONS

Kim *et al.* (1993) have argued that the solar neutrino problem can be explained by "condensed matter effects" on the distribution functions of fusing stellar nuclei, the principal effect being that the density of nuclei is lower at the point of last scattering than it is at infinity.[3] Liouville's theorem shows that the effects considered by Kim *et al.* are already included in the standard treatment.

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## REFERENCES

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[3]Kim *et al.* also raise the possibility that the rates are affected by nonequilibrium effects. In the introductory section of their paper quoted in Section 1 of this paper, they add, "However our problem is more complicated, as thermal equilibrium may not be restored locally when kinetic energy is converted to potential energy." That is, Kim *et al.* argue that there could be an effect if the gas of nuclei were driven away from equilibrium when one form of energy is converted into another form. However, such a departure from equilibrium would require a violation of the second law of thermodynamics. We have therefore not considered this possibility in detail.